

Fun spaces to classify

- Any set X endowed with the **trivial topology**, where the only open sets are \emptyset and X .
- Any set X endowed with the **discrete topology**, where all sets are open.
- The **Sierpiński space** $\{0, 1\}$ with topology $\{\emptyset, \{1\}, \{0, 1\}\}$.
- Any non-empty set X containing a point p endowed with the **particular point topology**, where the only open sets are the empty set and those sets that contain p .
- $[0, 1]$ with the standard topology.
- $(0, 1)$ with the standard topology.
- $[0, 1)$ with the standard topology (is this homeomorphic to \mathbb{R} ?).
- $[0, 1]$ with the **either-or topology**, where a set is open if it either does not contain $\{0.5\}$ or it does contain $(0, 1)$.
- The circle.
- \mathbb{N} with the **cofinite topology**, where a set is open if it is empty or if its complement is finite.
- \mathbb{R} with the **cocountable topology**, where a set is open if it is empty or if its complement is countable.
- The **topologist's sine curve**, $\{(x, \sin(1/x)) \mid x \in (0, 1]\} \cup \{(0, 0)\}$, with the subspace topology inherited from the Euclidean plane.
- The **Sorgenfrey line**, which is \mathbb{R} with the topology generated by the sets $[a, b)$.
- The **Sorgenfrey plane**, which is the product of the Sorgenfrey line with itself.

- The **long ray**, which is the cartesian product of the first uncountable ordinal ω_1 with $[0, 1)$. This set has the lexicographic order where $(a, b) < (c, d)$ if $a < c$ or if $a = c$ and $b < d$. It is endowed with the **order topology**, which is generated by the sets $\{x \mid a < x\}$ and $\{x \mid x < b\}$.
- The **long line**, which is constructed from the long ray in the same way that $(-1, 1)$ is constructed from $[0, 1)$.
- The **Tychonoff plank**, which is the product of the ordinal spaces $[0, \omega]$ and $[0, \omega_1]$.
- The **infinite broom**, which is the subspace of the Euclidean plane consisting of all closed line segments joining the origin to $(1, 1/n)$ for all positive integers n , together with the interval $(0.5, 1]$ on the x -axis.
- The **integer broom**, which is the subset of the Euclidean plane given by the polar coordinates $\{r = n \mid n \in \mathbb{N}_0\} \times \{\theta = 1/k \mid k \in \mathbb{N}, k \geq 1\} =: U \times V$. It has the product topology generated by the standard topology on V and the topology on U generated by the sets $\{n \mid a < n\}$.
- The **Cantor set**.
- The **Hilbert cube**, which is the topological product $\prod_{i=1}^{\infty} [0, 1/n]$.