Fun theorems

- In Fréchet (T_1) spaces, points are closed, and every subset is the intersection of all open sets containing that subset.
- All spaces are the quotient of some Hausdorff space.
- Compact sets are closed in Hausdorff spaces.
- If $f, g : X \to Y$ are continuous functions and Y is Hausdorff, then their equaliser is closed in X.
- Continuous functions into Hausdorff spaces are determined by their values on dense subsets.
- Completely regular spaces have their topology determined by their set of continuous functions to $\mathbb{R}$.
- Tychonoff spaces are exactly those spaces which can be embedded in compact Hausdorff spaces.
- Every space X is a subspace of some separable space with the same cardinality as X.
- Continuous functions from separable spaces to Hausdorff spaces are determined by countably many values.
- Separable metric spaces can be embedded in the Hilbert cube.
- X is sequential iff for all spaces Y and functions $f : X \to Y$, f is continuous iff for all sequences (x_n) and points x, $x_n \to x \Rightarrow f(x_n) \to f(x)$.
- The topology of second-countable spaces has cardinality less than or equal to that of the real line.
- Continuous images of compact spaces are compact.

- Any product of compact spaces is compact.
- Continuous images of ({}/path-/hyper-) connected spaces are ({}/path-/hyper-) connected.
- Every Hausdorff second-countable regular space is metrisable.
- For metrisable spaces, second-countability, separability, and Lindelöfness are equivalent, as are compactness, sequential compactness, countable compactness, and limit point compactness.
- For first-countable spaces, sequential compactness and countable compactness are equivalent.
- For Lindelöf spaces, compactness and countable compactness are equivalent.